

Constraining noncommutative field theories with holography

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An important window to quantum gravity phenomena in low energy noncommutative (NC) quantum field theories (QFTs) gets represented by a specific form of UV/IR mixing. Yet another important window to quantum gravity, a holography, manifests itself in effective QFTs as a distinct UV/IR connection. In matching these two principles, a useful relationship connecting the UV cutoff Λ_{UV} , the IR cutoff Λ_{IR} and the scale of noncommutativity Λ_{NC} , can be obtained. We show that an effective QFT endowed with both principles may not be capable to fit disparate experimental bounds simultaneously, like the muon $g - 2$ and the masslessness of the photon. Also, the constraints from the muon $g - 2$ preclude any possibility to observe the birefringence of the vacuum coming from objects at cosmological distances. On the other hand, in NC theories without the UV completion, where the perturbative aspect of the theory (obtained by truncating a power series in Λ_{NC}^{-2}) becomes important, a heuristic estimate of the region where the perturbative expansion is well-defined $E/\Lambda_{NC} \lesssim 1$, gets affected when holography is applied by providing the energy of the system E a Λ_{NC} -dependent lower limit. This may affect models which try to infer the scale Λ_{NC} by using data from low-energy experiments.

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Quantum field theories (QFTs) constructed on non-commutative (NC) spacetime:

$$[x_\mu, x_\nu] = i\theta_{\mu\nu}, \quad (1)$$

have received a great deal of interest lately mainly because of the possible appearance of such spacetimes in string theory [1–5]. In these QFTs noncommutativity is characterized by a real antisymmetric matrix $\theta_{\mu\nu}$ of dimension of length squared, which in the context of string theory reflects the properties of the background. At tree level a QFT formulated with (1) should switch to its commutative relative whenever the momenta of the field quanta are lowered below $\theta^{-1/2}$. In contrast, at loop level the inherent nonlocality of the full theory shows up in the UV/IR mixing phenomenon [6], meaning that switching to ordinary theories may occur at much lower momenta, depending on the ultimate UV cutoff in the theory. Figuratively speaking, the effect describes the linear growth of the size of a particle with its momentum, showing thus unambiguously its quantum-gravity origin [7].

The phenomenon of UV/IR mixing [6] is best understood by examining the behavior of the (nonplanar) loop graphs with the ordinary product of fields replaced by the Moyal star(★)-product (see e.g., [3, 4]). This results in phase factors depending on the virtual momenta of internal loops [8]. In a theory without UV completion ($\Lambda_{UV} \rightarrow \infty$) these phase factors, although efficient in damping out the high-energy part of the graphs, becomes together inefficient to control the vanishing momenta, i.e., the original UV divergences reappear as IR divergences. On the other hand, in presence of a finite Λ_{UV} no one sort of divergence will appear since the said phase factors effectively transform the highest energy scale (Λ_{UV}) into the lowest one (Λ_{IR}). The theory thus becomes an effective QFT with the UV and the IR cutoffs obeying a

relationship,

$$\Lambda_{UV}\Lambda_{IR} \sim \Lambda_{NC}^2, \quad (2)$$

where the scale of noncommutativity is heuristically introduced as $\Lambda_{NC}^{-2} \sim |\theta|$. To a good approximation the theory boils down to an ordinary (local) commutative theory where the crucial information from quantum gravity is brought in by the relationship (2).

The power of the UV/IR connection (2) is best seen heuristically by inspecting the photon self-energy, where explicit results in the NC approach in presence of a finite Λ_{UV} do exist both in the case of the external momentum above and below Λ_{IR} [9–11]. We know that in a commutative setting the integral is badly UV divergent if we regulate in the most naive way, giving [12] [27]

$$\Pi(k)^{\mu\nu} \propto g^{\mu\nu} \Pi(k) \sim g^{\mu\nu} \Lambda_{UV}^2. \quad (3)$$

From (2) one expects the same result to be valid also in the explicit NC approach for $k \gtrsim \Lambda_{IR}$, as confirmed in [11]. For $k \lesssim \Lambda_{IR}$ (2) is of course ineffective but in the deep IR regime one uses to invoke a standard quadratic decoupling of the massive species to get

$$\Pi(k) \sim (k^2/\Lambda_{IR}^2)\Lambda_{UV}^2. \quad (4)$$

For a generic non-SUSY theory, this agrees, by applying (2), with $\tilde{k}^2\Lambda_{UV}^4$ from [11]. When SUSY is softly broken $\Lambda_{UV}^4 \rightarrow \Delta M_{SUSY}^2\Lambda_{UV}^2$ [9–11], where ΔM_{SUSY}^2 is the supertrace of the mass matrix.

In the present paper we shall impose on the above QFT an extra requirement from the realm of quantum gravity: the holographic principle. Then we would like to test such a ‘beefed up’ theory against experimental data. We show that the latter requirement shows up as

an additional and distinct UV/IR correspondence. We find that additional restrictions imposed by holography become insurmountable barriers when fitting disparate experimental data.

For an effective QFT in a box of size L (providing an IR cutoff $\sim \Lambda_{\text{IR}}^{-1}$) the entropy scales extensively, $S_{\text{QFT}} \sim L^3 \Lambda_{\text{UV}}^3$, and therefore there is always a sufficiently large volume for which S_{QFT} would exceed the absolute Bekenstein-Hawking bound $S_{\text{BH}} \sim L^2 M_{\text{Pl}}^2$, where M_{Pl} is the Planck mass. Thus, considerations for the maximum possible entropy suggest that ordinary QFT may not be valid for arbitrarily large volumes, unless the UV and the IR cutoffs obey a constraint, $L \Lambda_{\text{UV}}^3 \lesssim M_{\text{Pl}}^2$. However, at saturation, this bound means that an effective QFT should also be capable to describe systems containing black holes, since it necessarily includes many states with Schwarzschild radius R_S much larger than the box size. There are however arguments for why an effective QFT appears unlikely to provide an adequate description of any system containing black holes [14, 15]. So, ordinary QFT may not be valid for much smaller volumes, but would apply provided a more stringent constraint is obeyed [16]

$$\Lambda_{\text{UV}}^3 \Lambda_{\text{IR}}^{-3} \lesssim M_{\text{Pl}}^{3/2} \Lambda_{\text{IR}}^{-3/2} \sim S_{\text{BH}}^{3/4}. \quad (5)$$

For a NC theory described by (2) and obeying the holographic requirement (5), additional constraints can be derived

$$\Lambda_{\text{IR}} \gtrsim \Lambda_{\text{NC}} \left(\frac{\Lambda_{\text{NC}}}{M_{\text{Pl}}} \right)^{1/3} \quad (6)$$

$$\Lambda_{\text{UV}} \lesssim \Lambda_{\text{NC}} \left(\frac{M_{\text{Pl}}}{\Lambda_{\text{NC}}} \right)^{1/3} \quad (7)$$

which represent, in addition to (2), a further connection between the parameters describing the field theory (Λ_{UV} and Λ_{IR}) and that describing spacetime (Λ_{NC})

One may however justifiably object that our bounds (5-7), as they stand, do not reflect possible modifications due to NC black hole thermodynamics. Following [17, 18], such effects indeed do arise due to the fuzziness of space induced by the space-space component of the uncertainty relation (1). Consequently, various thermodynamic entities, like the mass of the black hole and the Schwarzschild radius get modified in a spherically symmetric, stationary NC Schwarzschild spacetime [17, 18]. The Bekenstein-Hawking entropy S_{BH} also receives corrections when $\theta \neq 0$. For fixed θ and an arbitrary-sized black hole, the Schwarzschild radius in NC settings cannot be obtained in a closed form, but useful relations can be obtained in the large radius regime $R_S^2/4\theta \gg 1$. It was shown [17] that neglecting the usual logarithmic correction at $\theta = 0$ (otherwise unimportant in our case), and at the leading order in the parameter $R_S^2/4\theta$, both the NC horizon area A and the NC black hole entropy follow the same functional change as a function of θ , such

that the exact functional form of the (usual) commutative area law, $S_{\text{BH}}^{\text{NC}} = AM_{\text{Pl}}^2/4$, stays preserved. This means that NC thermodynamical laws are a NC deformation of the usual laws. Thus our bounds (5-7) are unaffected in NC settings. One can furthermore check whether the above field-theoretical setup fits within the given regime $R_S^2/4\theta \gg 1$. One can easily show that the constraint $R_S^2/4\theta \gg 1$, with the aid of Eq. (2), boils down to $\Lambda_{\text{UV}}/\Lambda_{\text{IR}} \gg (L/R_S)^2$. The *rhs* of the latter constraint is always $\gtrsim 1$, by noting (as discussed before) that any effective field theory approach should not describe states already collapsed to a black hole. Thus, we are left with $\Lambda_{\text{UV}}/\Lambda_{\text{IR}} \gg 1$, which is nothing but the consistency constraint for any ordinary QFT.

Equipped with these relationships we seek for further constraints on Λ_{UV} , Λ_{IR} and Λ_{NC} by testing the theory against experimental data. First we seek for further constraints on Λ_{UV} , Λ_{IR} and Λ_{NC} by considering the muon $g - 2$. Of essence here is to notice that a contribution from radiative corrections which otherwise would tend to zero when both $\Lambda_{\text{UV}} \rightarrow \infty$, $\Lambda_{\text{IR}} \rightarrow 0$, now yields a finite answer because Λ_{UV} and Λ_{IR} should obey a non-trivial constraint from the UV/IR mixing (2) as well as the holographic constraints (5). We have,

$$\Delta(g_\mu - 2) \sim \frac{\alpha}{\pi} \left[\left(\frac{m_\mu}{\Lambda_{\text{UV}}} \right)^2 + \left(\frac{\Lambda_{\text{IR}}}{m_\mu} \right)^2 \right]. \quad (8)$$

Since we are no more able to pick out Λ_{UV} independently from Λ_{IR} the contribution (8) becomes nonzero. Obviously, this correction of the quantum-gravitational origin greatly surpasses the usual Planck-scale correction.

By applying (2) and assuming first $\Lambda_{\text{NC}} \gtrsim m_\mu$ one arrives at

$$\Delta(g_\mu - 2)_{\text{IR}} \sim \frac{\alpha}{\pi} \left(\frac{\Lambda_{\text{IR}}}{m_\mu} \right)^2. \quad (9)$$

From the report of the muon E821 anomalous magnetic moment measurements at BNL [19] we know that

$$\frac{g_\mu - 2}{2} (\text{Exp} - \text{SM}) = (22 - 26) \times 10^{-10}. \quad (10)$$

In turn, this and the holographic constraint (6) on Λ_{IR} implies

$$m_\mu \lesssim \Lambda_{\text{NC}} \lesssim 0.1 \text{ TeV}. \quad (11)$$

Also one obtains $\Lambda_{\text{IR}} \lesssim 10^{-1} \text{ MeV}$ and $10^5 \text{ MeV} \lesssim \Lambda_{\text{UV}} \lesssim 10^5 \text{ TeV}$.

In the opposite regime, $\Lambda_{\text{NC}} \lesssim m_\mu$, one gets

$$\Delta(g_\mu - 2)_{\text{UV}} \sim \frac{\alpha}{\pi} \left(\frac{m_\mu}{\Lambda_{\text{UV}}} \right)^2, \quad (12)$$

from where the constraint (7) and Eq. (10) give

$$10^{-4} \text{ MeV} \lesssim \Lambda_{\text{NC}} \lesssim m_\mu, \quad (13)$$

together with $\Lambda_{UV} \gtrsim 10^2$ GeV and 10^{-1} MeV $\gtrsim \Lambda_{IR} \gtrsim 10^{-13}$ MeV. Note that our constraints (11) and (13) are the exceptional ones, providing for the first time the upper bound for the scale of noncommutativity,

$$\Lambda_{NC} \lesssim \mathcal{O}(0.1) \text{ TeV}. \quad (14)$$

The other NC approaches (without UV completion and holography, whether based on the truncated theory or not) when confronting experiments may yield only a lower limit on Λ_{NC} . The best limits yielded $\Lambda_{NC} \gtrsim \mathcal{O}(100)$ TeV [20]. This striking mismatch could signal how strongly may the outcome be influenced by the NC UV/IR mixing and holography. Also, there is an illuminating argument [21] that when calculating loop effects any NC approach based on the truncated- θ expansion can reliably approximate the full theory only in the region below Λ_{IR} , where the result crucially depends on the UV completion of our model, i.e., the region where our effective theory is deprived of predictive power.

Note that lowering the UV scale down to the dark energy scale of the universe at present, $\Lambda_{UV} \sim 10^{-3}$ eV, is not allowed by our constraints (11) and (13). The present theoretical setup is hence not capable to shed light on the dark energy/cosmological constant problem [22].

In the following we show how strong theoretical and experimental constraints as outlined above would actually make our NC setup incapable to account for other experimental data. We shall concentrate on the UV/IR mixing in the polarization tensor for a pure U(1) NC gauge theory, [10, 11]. When SUSY is softly broken, a nontrivial dispersion relation induces a sizable Lorentz symmetry violating mass term for the photon, if the photon momentum $k \gtrsim \Lambda_{IR}$ [11]. Since masslessness of the photon is well tested up to at least 1 GeV, one requires

$$\Lambda_{IR} \gtrsim 1 \text{ GeV}, \quad (15)$$

which is strikingly inconsistent with both cases (11) and (13). In a different regime, $\Lambda_{IR} \gtrsim 1$ TeV, the same dispersion relation would induce the birefringence effect for gamma rays having energies ~ 1 TeV, and reaching us from astrophysical sources at the cosmological distances. Such an effect would alter the light speed to $\approx c(1 - \Delta n)$. Again, the constraints from the muon $g - 2$ preclude any possibility for the NC-induced birefringence for high energy gamma rays. Still, one may be curious to see the impact of our basic constraints (2), (6) and (7) on the said birefringence effect. Employing the strongest limit on Δn to date [23], attaining the level of 10^{-37} and coming from the recent measurements of linear polarization in gamma rays from the two gamma-ray bursts at $z \gtrsim 0.1$, one obtains [11]

$$(10^{18} \text{ GeV})^2 \Lambda_{UV}^2 \Lambda_{NC}^{-4} \lesssim 10^{-3}. \quad (16)$$

Thus, based solely on the birefringence constraint, one sees, from Eqs. (2), (6) and (7), that the Planck-scale noncommutativity, $\Lambda_{UV} \sim \Lambda_{NC} \gtrsim M_{Pl}$, is preferred.

Although (as already noted above) the truncated θ -expansion is not likely to be a reliable approximation of the full theory when considering loop effects, i.e. considering the expansion in $k\theta\ell$ with $k(\ell)$ playing the role of external (loop) momentum, there have been, on the other hand, a great deal of attempts undertaken to constrain Λ_{NC} by using scattering tree-level processes within such an approach [24]. In that case the truncated θ -expanded theory can give a good approximation of the full theory if heuristically $E/\Lambda_{NC} \lesssim 1$, where E is the energy of the system. The less E/Λ_{NC} , the better the reliability of the approximation. The holographic restrictions (6) and (7) can however give a consequential constraint to this statement by providing E a lower limit. Indeed, $E \gtrsim \Lambda_{IR}$ requires

$$E \gtrsim \Lambda_{NC} \left(\frac{\Lambda_{NC}}{M_{Pl}} \right)^{1/3}, \quad (17)$$

by (6). For instance, if the inferred Λ_{NC} (obtained by comparing a given scattering process to the data) is around 10 TeV, then $E \gtrsim 100$ MeV. This obviously can affect placing limits on Λ_{NC} when using high-precision low-energy experiments, notably with atomic constraints on Λ_{NC} [25].

In conclusion, we have shown that information from two distinct realms of quantum gravity may not be readily manifested when imposed on an effective QFT. In a sense, the UV/IR mixing from noncommutativity and holography do not go hand in hand when implemented in the field-theoretical approach. Although an ultimate UV cutoff is expected to be capable of providing a good approximation to the effect of UV completion, owing to the string theory realization of both noncommutativity/holography, still a proven lack of universality in NC theories [6, 26] may cause a failure of the theory to fit simultaneously disparate experimental constraints. Or simply an information from two distinct UV/IR mixings, (2) and (5) respectively, when implemented on models at low energy scales, becomes more overabundant and scrambled than with the either UV/IR mixing taken separately.

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